

A manipulator is said to have redundancy if it has more degrees of freedom than are necessary to perform a given task. In this chapter, after a discussion about the advantages and disadvantages of redundant manipulators, a method of actively utilizing such redundancy will be given. This method will be applied to avoiding obstacles and avoiding singular configurations, and a numerical calculation method that is effective in the above approach will be described.

7.1 Redundant Manipulators

A human arm can be considered to have seven degrees of freedom. A hand requires a total of six degrees of freedom (three to hold an object at a certain position in space, and three more to orient it); therefore, the human arm has one redundant degree of freedom for this fundamental task of positioning and orienting the grasped object. This explains why we can move the elbow freely while grasping an object steadily in a certain position and orientation. This excess degree of freedom is the redundancy, and it gives the human arm great versatility and broad applicability.

The same argument applies to manipulators. Since six degrees of freedom are enough for full position and orientation control of an end effector, a manipulator with seven degrees of freedom can be said to be redundant. Even manipulators with six or fewer degrees of freedom are often regarded to have redundancy because the particular task to be performed requires fewer degrees of freedom than the manipulator has available. This is one of the main reasons why we need to consider the issue of redundancy.

What are the advantages of redundant manipulators? Generally speaking, as in the case of the human arm, they excel in versatility and applicability. More specifically, they have the potential to avoid singularities, avoid obstacles, avoid structural limitations (e.g., angle limits of a rotational joint), carry out reasonable actions (e.g., low-energy-consuming motion, balanced motion among individual joint velocities, balanced configuration among joint driving forces), reach behind an object, crawl into concaves, and so on. Redundancy can also be used to make a manipulator more reliable in the sense that it can perform certain tasks even after a failure of some joints. This is important in certain application fields, such as space and the deep sea.

Of course, redundant manipulators have disadvantages too. They have more joints and actuators. Their structure is more complex, bulkier, and

heavier. More complicated control algorithms are required, so the amount of necessary computation increases. Redundant manipulators will not be truly beneficial unless these disadvantages are offset by some advantages.

7.2 Task-Decomposition Approach

7.2.1 Decomposing a Task into Subtasks with Priority Order^{1,2}

Most complicated tasks given to a manipulator can be formulated in such a way that the task is broken down into several subtasks with a priority order. Each subtask is performed using the degrees of freedom that remain after all the subtasks with higher priority have been implemented. A welding task, for instance, might be broken down into hand-position control and hand-orientation control, with hand positioning given first priority. A task of tracking a trajectory through a workspace with obstacles might be decomposed into controlling the position and orientation of the hand and avoiding obstacles, with the former given first priority. It is obvious that control problems of redundant manipulators can be approached via the subtask method by regarding a task to be done by a redundant manipulator as the subtask with first priority and regarding the reason for using the redundancy (singularity avoidance, joint limit avoidance, etc.) as the subtask with second priority.

We assume that each subtask is given in the form of either a desired trajectory of variables suitable for describing the subtask, or a certain criterion. The first form is suitable when the operator can give a trajectory that achieves the given subtask. The second form is used when the operator does not know the actual desired trajectory but knows how to evaluate the trajectory; for example, when the subtask is to keep the manipulator configuration away from singularities or to keep the joint angles within their limits, the configuration is evaluated using a criterion.

For ease of explanation, the discussion will be limited to simple cases in which the task is broken down into two subtasks and the first subtask is given in the form of a desired trajectory.

7.2.2 Basic Equations

Consider a manipulator with n degrees of freedom. The joint variable of the i th joint is q_i ($i = 1, 2, \dots, n$). The manipulator configuration is denoted by the vector $\mathbf{q} = [q_1, q_2, \dots, q_n]^T$.

Assume that the first subtask can be described properly by an m_1 -dimensional vector, y_1 , which is a function of q :

$$y_1 = f_1(q). \quad (7.1)$$

Assume also that the desired trajectory for y_1 is given: $y_{1d}(t)$ ($0 \leq t \leq t_f$; t_f is a terminal time). A vector (such as y_1) which is suitable for describing a manipulation task is called a *manipulation vector*.

For the second subtask, consider the following two cases:

Case 1 An m_2 -dimensional manipulation vector y_2 is given by

$$y_2 = f_2(q), \quad (7.2)$$

and the second subtask is specified by the desired trajectory $y_{2d}(t)$ ($0 \leq t \leq t_f$) for y_2 .

Case 2 A criterion function

$$p = V(q) \quad (7.3)$$

is given, and the second subtask is to keep this criterion as large as possible.

The problem we consider is first to obtain the joint velocity that achieves the first subtask of realizing $y_1 = y_{1d}$ and then, if there is any ability left, to perform as much of the second subtask as possible at each time instant. The dynamics of the manipulator are not considered; that is, the problem is treated as purely kinematic. It is possible to formulate the problem in both case 1 and case 2 as an instantaneous optimal control problem in a mathematically more rigorous way, as we will do in subsection 7.2.5; however, we will first solve the problem in a rather heuristic way.

The general solution of the joint velocity that performs the first subtask will be given. Differentiating equation 7.1 with respect to time yields

$$\dot{y}_1 = J_1 \dot{q}, \quad (7.4)$$

where $J_j = df_j(q)/dq^T$ ($j = 1, 2$) is the Jacobian matrix of y_j with respect to q . When the desired trajectory y_{1d} is given, from equation A2.22 in appendix 2, the general solution for \dot{q} of equation 7.4 is

$$\dot{q} = J_1^+ \dot{y}_{1d} + (\mathbf{I} - J_1^+ J_1) k_1, \quad (7.5)$$

where k_1 is an n -dimensional arbitrary constant vector.³ The first term on the right-hand side of equation 7.5 is the joint velocity to achieve the desired

trajectory, $y_{1d}(t)$. When there are multiple solutions for \dot{q} satisfying equation 7.4, this term gives a solution that minimizes $\|\dot{q}\|$, the Euclidean norm of \dot{q} . Also, even when there is no solution that satisfies equation 7.4, this term provides an approximate solution that minimizes the norm $\|\dot{y}_{1d} - J_1 \dot{q}\|$, which is a kind of measure for approximation error. The second term on the right-hand side of equation 7.5 represents the redundancy left after performing the first subtask. Equation 7.5 is the basic equation for using a redundancy.

7.2.3 Second Subtask Given by Desired Trajectory

In this subsection we will consider case 1, where the second subtask is specified by the desired trajectory $y_{2d}(t)$ of the manipulation variable y_2 . What we have to do is select k_1 in equation 7.5 so as to realize y_{2d} as closely as possible. Differentiating equation 7.2, we have

$$\dot{y}_2 = J_2 \dot{q}. \quad (7.6)$$

Substituting $\dot{y}_2 = \dot{y}_{2d}$ and equation 7.5 into 7.6, we obtain

$$\dot{y}_{2d} - J_2 J_1^+ \dot{y}_{1d} = J_2 (\mathbf{I} - J_1^+ J_1) k_1. \quad (7.7)$$

Letting $J_2 = J_2 (\mathbf{I} - J_1^+ J_1)$ and using equation A2.22, we obtain from equation 7.7

$$k_1 = J_2^+ (\dot{y}_{2d} - J_2 J_1^+ \dot{y}_{1d}) + (\mathbf{I} - J_2^+ J_2) k_2, \quad (7.8)$$

where k_2 is an n -dimensional arbitrary constant vector. Note that the relation

$$(\mathbf{I} - J_1^+ J_1) J_2^+ = J_2^+ \quad (7.9)$$

holds (see ref. 4 and exercise 7.1). Therefore, from equations 7.5, 7.8, and 7.9,

$$\dot{q}_d = J_1^+ \dot{y}_{1d} + J_2^+ (\dot{y}_{2d} - J_2 J_1^+ \dot{y}_{1d}) + (\mathbf{I} - J_1^+ J_1 - J_2^+ J_2) k_2 \quad (7.10a)$$

is the desired joint velocity that first realizes trajectory y_{1d} and then realizes y_{2d} as closely as possible using the remaining redundancy.

If $(\mathbf{I} - J_1^+ J_1 - J_2^+ J_2)$ is not zero, then some further redundancy is still left to perform a third subtask. When $y_2 = q$ (that is, when the second subtask provides the desired trajectory of the whole arm configuration), since $J_2 = \mathbf{I}$ we have

$$J_2^+ = (\mathbf{I} - J_1^+ J_1)^+ = (\mathbf{I} - J_1^+ J_1).$$

Hence, equation 7.10a simplifies to

$$\dot{\mathbf{q}}_d = \mathbf{J}_1^+ \dot{\mathbf{y}}_{1d} + (\mathbf{I} - \mathbf{J}_1^+ \mathbf{J}_1) \dot{\mathbf{y}}_{2d}, \quad (7.10b)$$

in which the term with \mathbf{k}_2 disappears.

When the servo loop of each joint axis functions perfectly to produce the desired joint velocity, $\mathbf{y}_{1d}(t)$ is followed exactly. Some deviation often occurs, however, between the desired subtask trajectory $\mathbf{y}_{2d}(t)$ and the actually realized trajectory $\mathbf{y}_2(t)$, because of the insufficient number of remaining degrees of freedom. One way to cope with such a situation is to modify the desired velocity $\dot{\mathbf{y}}_{2d}$ to take the deviation into consideration. For example, we can define the modified velocity $\dot{\mathbf{y}}_{2d}^*$ by

$$\dot{\mathbf{y}}_{2d}^* = \dot{\mathbf{y}}_{2d} + \mathbf{H}_2(\mathbf{y}_{2d} - \mathbf{y}_2), \quad (7.11)$$

where \mathbf{H}_2 is an appropriate diagonal gain matrix, and then replace $\dot{\mathbf{y}}_{2d}$ in equation 7.10a or 7.10b with $\dot{\mathbf{y}}_{2d}^*$.

7.2.4 Second Subtask Given by Criterion Function

In this subsection we will consider case 2, where the second subtask is specified by a criterion function. As in case 1, we select the value \mathbf{k}_1 in equation 7.5 so as to make the criterion p given by equation 7.3 as large as possible. One natural approach is to determine \mathbf{k}_1 by the following equations (ref. 3):

$$\mathbf{k}_1 = \xi \mathbf{k}_p, \quad (7.12)$$

$$\xi = [\xi_1, \xi_2, \dots, \xi_n]^T, \quad (7.13)$$

$$\xi_i = \partial V(\mathbf{q}) / \partial q_i, \quad (7.14)$$

where \mathbf{k}_p is an appropriate positive constant. The desired joint velocity, $\dot{\mathbf{q}}_d$, is given by

$$\dot{\mathbf{q}}_d = \mathbf{J}_1^+ \dot{\mathbf{y}}_{1d} + (\mathbf{I} - \mathbf{J}_1^+ \mathbf{J}_1) \xi \mathbf{k}_p. \quad (7.15)$$

Hence, we have

$$\dot{p} = \xi^T \mathbf{J}_1^+ \dot{\mathbf{y}}_{1d} + \xi^T (\mathbf{I} - \mathbf{J}_1^+ \mathbf{J}_1) \xi \mathbf{k}_p. \quad (7.16)$$

Since $(\mathbf{I} - \mathbf{J}_1^+ \mathbf{J}_1)$ is nonnegative definite, the second term on the right-hand side of equation 7.16 is always nonnegative, causing the value of criterion p to increase.⁵

Mathematically, the vector \mathbf{k}_1 of equation 7.12 is the steepest gradient vector of the function $V(\mathbf{q})$; the vector $(\mathbf{I} - \mathbf{J}_1^+ \mathbf{J}_1) \xi \mathbf{k}_p$ corresponds to the orthogonal projection of \mathbf{k}_1 on the null space of \mathbf{J}_1 . The constant \mathbf{k}_p is chosen so as to make p increase as quickly as possible under the condition that $\dot{\mathbf{q}}_d$ does not become excessively large.

7.2.5 Formulation as Instantaneous Optimization Problem

The problem formulation in subsection 7.2.2 and the solutions in subsections 7.2.3 and 7.2.4 are rather intuitive. In this subsection it is shown that these results can be obtained as the rigorous solutions to some instantaneous optimization problems by adopting an appropriate criterion function for the second subtask.

For case 1, let the criterion function for the second subtask be

$$p_1 = \|\dot{\mathbf{y}}_{2d} - \dot{\mathbf{y}}_2 + \mathbf{H}_2(\mathbf{y}_{2d} - \mathbf{y}_2)\|. \quad (7.17)$$

The optimal solution, which minimizes equation 7.17 at each time instant under the constraint 7.5, is given by equations 7.10a and 7.11.

For case 2, using the original criterion $V(\mathbf{q})$ of equation 7.3, let the criterion function for the second subtask be

$$p_2 = k_p \frac{dV(\mathbf{q})}{dt} - \frac{1}{2} \|\dot{\mathbf{q}} - \mathbf{J}_1^+ \dot{\mathbf{y}}_{1d}\|^2. \quad (7.18)$$

The optimal solution maximizing equation 7.18 at each time instant under the constraint 7.5 is given by equation 7.15. The first term of equation 7.18 attempts to increase the original criterion; the second term tries to minimize the difference between $\dot{\mathbf{q}}$ and the minimal norm solution $\mathbf{J}_1^+ \dot{\mathbf{y}}_{1d}$ of the first subtask. The coefficient \mathbf{k}_p represents the weight of importance of the first effort compared to the second effort.

7.3 Application to Avoiding Obstacles and Singularities

7.3.1 Avoiding Obstacles

Consider the three-link manipulator shown in figure 7.1, which moves in the X - Y plane. The required task is to make the end effector follow a desired trajectory while avoiding collisions with the obstacle shown as the hatched area in the figure. We will solve this problem of avoiding the obstacle as an example of case 1 (see ref. 2).

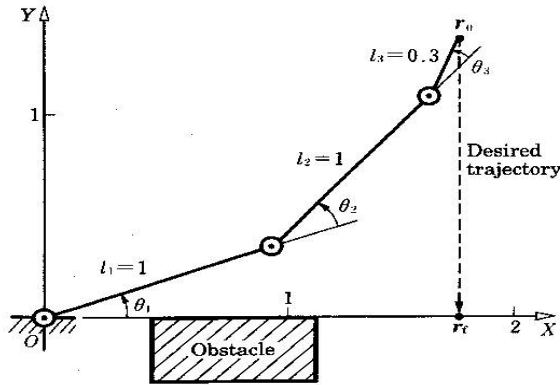


Figure 7.1
Three-link manipulator.

The link lengths are $l_1 = l_2 = 1$ and $l_3 = 0.3$, and there is no hardware limit on the rotation of any joint. At the beginning of the task, the initial configuration at time $t = 0$ is given by

$$q_0 = [20^\circ, 30^\circ, 20^\circ]^T,$$

which corresponds to

$$r_0 = [x_0, y_0]^T \cong [1.69, 1.39]^T.$$

The end effector is required to move from initial position r_0 to final position $r_f = [x_0, 0]^T$ by time $t = 1$, along a straight line parallel to the Y axis. At the same time, the manipulator should avoid colliding with the hatched obstacle zone. This task is divided into a first subtask (moving along the desired trajectory) and a second subtask (avoiding the obstacle).

For the manipulation variable y_1 of the first subtask, we use the end-effector position r :

$$y_1 = \begin{bmatrix} l_1 C_1 + l_2 C_{12} + l_3 C_{123} \\ l_1 S_1 + l_2 S_{12} + l_3 S_{123} \end{bmatrix}. \quad (7.19)$$

Hence, the Jacobian matrix J_1 is given by

$$J_1 = \begin{bmatrix} -l_1 S_1 - l_2 S_{12} - l_3 S_{123} & -l_2 S_{12} - l_3 S_{123} & -l_3 S_{123} \\ l_1 C_1 + l_2 C_{12} + l_3 C_{123} & l_2 C_{12} + l_3 C_{123} & l_3 C_{123} \end{bmatrix}. \quad (7.20)$$

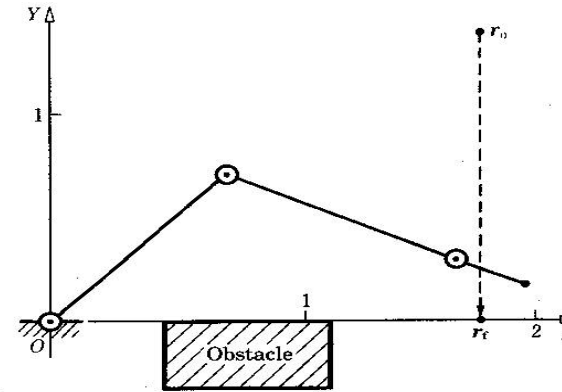


Figure 7.2
Obstacle and reference configuration taught for obstacle avoidance.

If we choose the desired trajectory using equation 5.7, then we have

$$y_{1d}(t) = \begin{bmatrix} x_0 \\ y_0 - (3 - 2t)t^2 y_0 \end{bmatrix}, \quad 0 \leq t \leq 1. \quad (7.21)$$

For the second subtask, we assume that a constant reference arm configuration, suitable for obstacle avoidance, has already been taught (say, by a human operator). Hence, the second subtask can be specified as trying to come as close to this configuration as possible. This means that the manipulation variable y_2 for the second subtask is given by the joint vector q . We assume that the reference arm configuration that plays the role of the desired trajectory for the second subtask has been given as

$$y_{2d} = \begin{bmatrix} 45^\circ \\ -70^\circ \\ 0^\circ \end{bmatrix}. \quad (7.22)$$

This reference configuration for collision avoidance is shown in figure 7.2.

Now that we have defined our terms, from equations 7.10b and 7.11 the desired joint velocity \dot{q}_d is given by

$$\dot{q}_d = J_1^+ \begin{bmatrix} 0 \\ -6(1-t)y_0 \end{bmatrix} + (I_3 - J_1^+ J_1) H_2 (y_{2d} - y_2). \quad (7.23)$$

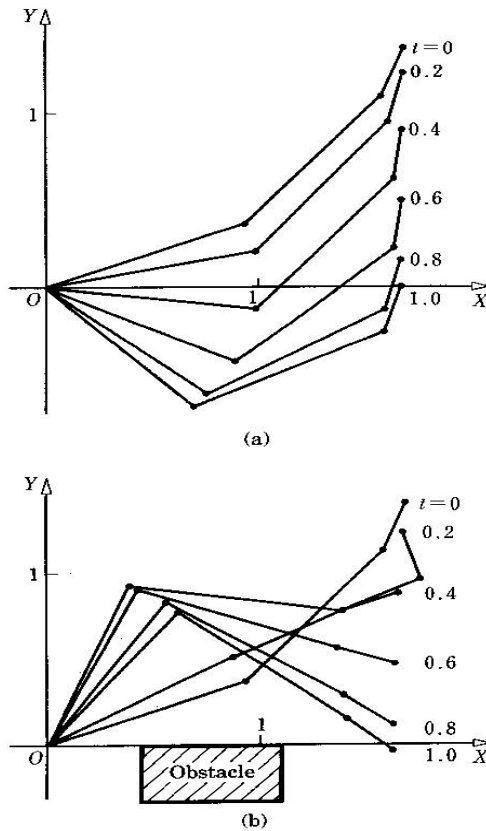


Figure 7.3
Simulation of obstacle avoidance. (a) Without using redundancy ($H_2 = 0$). (b) Using redundancy ($H_2 = 0.2I_2$).

(The second term on the right hand side of this equation is needed because of the inconsistency between y_{2d} and $y_{1d}(t)$.)

The simulation results shown in figure 7.3 were obtained by assuming that the desired joint velocity of equation 7.23 occurs perfectly. Figure 7.3a depicts the case when $H_2 = 0$, so \dot{q} has been chosen to minimize $\|\dot{q}\|$, ignoring the obstacle avoidance. Figure 7.3b depicts the case when $H_2 = 0.2I_2$ (that is, when the redundancy is utilized). It is clear from the figure that collision with the obstacle has been avoided by the use of redundancy in the simulated case.

The above example treated a very simple obstacle. This approach to avoiding obstacles could also be adopted to cope with more complicated obstacles or time-varying obstacles by using several reference configurations taught beforehand and switching them in turn.

Another approach is to define a criterion function to express the proximity between the manipulator and obstacles⁶ and to make it the criterion for the second subtask (ref. 2). This approach does not need any reference configuration, although it requires detailed information about the obstacles and a large amount of calculation.

7.3.2 Avoiding Singularities

As an example of case 2, where the second subtask is specified by a criterion function, let us consider the problem of making the end effector track a desired trajectory while avoiding singular configurations as much as possible (ref. 5).

A manipulator generally has certain configurations in which it can no longer move its end effector to change position or orientation in certain directions. These configurations are called singular configurations, as was discussed in subsection 2.5.6. It is undesirable not only for a manipulator to fall into such a singular configuration but also for it even to come close to one. Some joint velocities would become excessively large trying to maintain the desired trajectory. The manipulability measure was discussed in section 4.2 as a quantitative measure of the capability of a manipulator to move its end effector freely in any direction. This measure can also be regarded as an index of the distance from singular configurations. Hence, the manipulability measure can be used as the criterion when the second subtask is the avoidance of singularities.

Consider the same three-link manipulator as in the preceding subsection. The end-effector position r is again taken as the first manipulation variable,

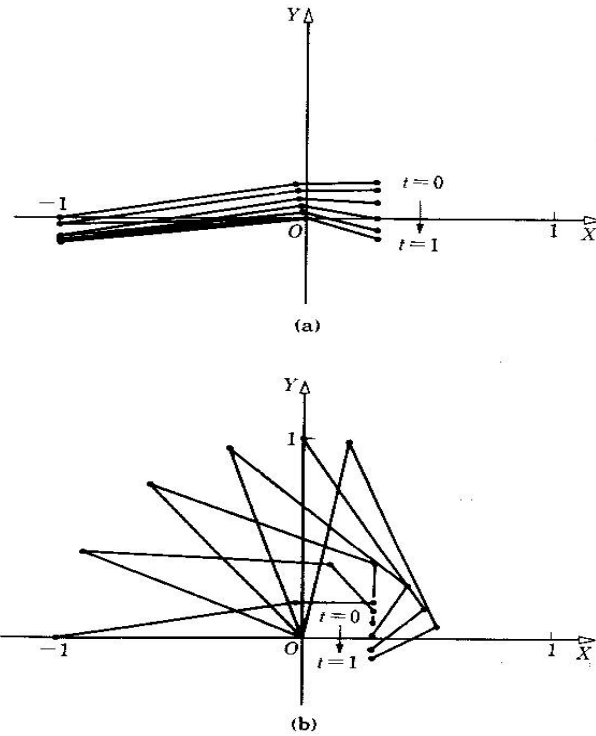


Figure 7.4 Simulation of singularity avoidance. (a) Without using redundancy ($k_p = 0$). (b) Using redundancy ($k_p = 20$).

y_1 . The manipulability measure for r is taken as the criterion for the second subtask. Hence, using equation 4.8 and the Jacobian matrix J_1 of equation 7.20, we have

$$V(q) = \sqrt{\det J_1 J_1^T}$$

Therefore, ξ_l of equation 7.14 is given by

$$\xi_l = \frac{1}{2} \sqrt{\det J_1 J_1^T} \sum_{i,j=1}^2 q_{ij} (J_{1ii} J_{1j}^T + J_{1ji} J_{1i}^T), \quad l = 1, 2, 3 \quad (7.24)$$

where q_{ij} is the (i, j) element of the inverse matrix of $(J_1 J_1^T)$, J_{1i} is the i th row vector of J_1 , and J_{1ii} is the partial differential coefficient of J_{1i} with respect to θ_i . The vector ξ in equation 7.15 is obtained from equation 7.24.

Some results of computer simulation are shown in figure 7.4. The initial configuration is $q_0 = [180^\circ, 170^\circ, -10^\circ]^T$, which corresponds to the end-effector position $r_0 = [x_0, y_0]^T = [0.28, 0.17]^T$. The desired trajectory is given by

$$y_{1d}(t) = \begin{bmatrix} x_0 \\ y_0 - (3 - 2t)t^2(y_0 + 0.1) \end{bmatrix}, \quad 0 \leq t \leq 1 \quad (7.25)$$

which is a straight line parallel to the Y axis going from r_0 to the final position $r_f = [x_0, -0.1]^T$. Equation 7.25 is, again, obtained from equation 5.7. Figure 7.4a shows the change of arm configuration for the case of $k_p = 0$ in equation 7.15; that is, the case where the simple pseudo-inverse control law is applied and no attempt is made to avoid singularity. Figure 7.4b is for the case where $k_p = 20$; the singularity is avoided by trying to keep the manipulability measure large. Figure 7.5 shows the change of manipula-

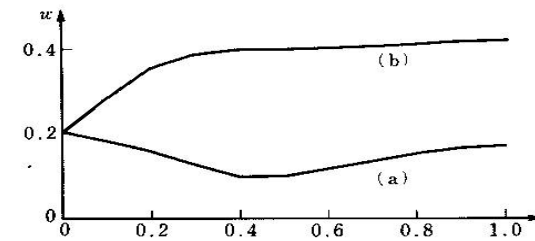


Figure 7.5 Trajectories of manipulability measure. (a) Without using redundancy ($k_p = 0$). (b) Using redundancy ($k_p = 20$).

bility measure in cases a and b of figure 7.4. As can be seen from the figure, the utilization of redundancy is quite effective in avoiding singularity.

7.4 Computational Method for Desired Joint Velocity

One of the difficulties in applying the method described above is the large amount of computation needed for obtaining the pseudo-inverse matrix. In this section, we will develop an efficient computational method for obtaining \dot{q} satisfying

$$\dot{q} = J^+ \dot{y}_d + (\mathbf{I} - J^+ J)k, \quad (7.26)$$

which corresponds to equation 7.5, for given values of J , \dot{y}_d , and k .

If the matrix (JJ^T) is nonsingular, \dot{q} can be obtained by the following procedure⁷: First, obtain the solution α of

$$(JJ^T)\alpha = \dot{y}_d - Jk \quad (7.27)$$

by an appropriate method for solving linear algebraic equations, such as Gauss elimination. Then, calculate \dot{q} by

$$\dot{q} = J^T \alpha + k. \quad (7.28)$$

It is easy to check that \dot{q} calculated this way is equal to \dot{q} given by equation 7.26, since $J^+ = J^T(JJ^T)^{-1}$.

At singular configurations, however, (JJ^T) becomes singular and the solution \dot{q} to equation 7.26 is not necessarily continuous with respect to the elements of J . Because of the relationship of \dot{q} to q through J , \dot{q} is also not necessarily continuous with respect to q . One way to cope with this problem is to apply regularization, so that (JJ^T) is replaced by $(JJ^T + k_s \mathbf{I})$ near singularities.⁸ Thus, in place of equation 7.27 we use

$$(JJ^T + k_s \mathbf{I})\alpha = \dot{y}_d - Jk, \quad (7.29)$$

where k_s is a scalar continuous function of q that is positive near singularities and zero for other q . For example, using the manipulability measure

$w = \sqrt{\det JJ^T}$, k_s can be assigned to be

$$k_s = \begin{cases} k_0(1 - w/w_0)^2, & w < w_0 \\ 0, & w \geq w_0 \end{cases} \quad (7.30)$$

where k_0 and w_0 are appropriate positive constants. The parameter k_0

specifies the magnitude of regularization, and w_0 represents the threshold separating the vicinity of singularities from the other region.

Exercises

7.1 Show that the equality $B(CB)^+ = (CB)^+$ holds for any $n \times n$ symmetric and idempotent matrix B (i.e., $B^T = B$ and $BB = B$) and $m \times n$ matrix C (ref. 4). Using this relation, prove equation 7.9.

7.2 Show that the solution that minimizes equation 7.17 at each instant of time under the constraint 7.5 is given by equations 7.10a and 7.11.

7.3 Show that the solution that maximizes equation 7.18 at each instant of time under the constraint 7.5 is given by equation 7.15.

7.4 Calculate \dot{q} from equation 7.26 when

$$J = \begin{bmatrix} 0 & 2 & 1 & 2 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}, \quad \dot{y}_d = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad k = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$

Also calculate the same value using equations 7.27 and 7.28.

7.5 Show that \dot{q} obtained from equation 7.28 using α calculated from equation 7.29 is equal to \dot{q} minimizing

$$\|J\dot{q} - \dot{y}_d\|^2 + k_s \|\dot{q} - k\|^2.$$

7.6 Decompose the task of writing letters on a sheet of paper with a ballpoint pen into several subtasks with priority order, and state the reasons for that decomposition. Do the same with the task of carrying a cup full of liquid.

7.7 Assume that the three-link manipulator shown in figure 7.1 has the following joint angle limits:

$$\theta_{i \min} \leq \theta_i \leq \theta_{i \max}, \quad i = 1, 2, 3.$$

Describe a formula for the desired joint velocity that makes the endpoint of the manipulator follow a given desired trajectory while keeping the joint angles within their limits as much as possible.

References

1. H. Hanafusa, T. Yoshikawa, and Y. Nakamura, "Analysis and Control of Articulated Robot Arms with Redundancy," in *Proceedings of the 8th IFAC World Congress (1981)*, vol. XIV, pp. 78–83.
2. Y. Nakamura, H. Hanafusa, and T. Yoshikawa, "Task-Priority Based Redundancy Control of Robot Manipulators," *International Journal of Robotics Research* 6 (1987): 3–15.
3. A. Liégeois, "Automatic Supervisory Control of the Configuration and Behavior of Multi-body Mechanisms," *IEEE Transactions on Systems, Man, and Cybernetics* 7, no. 12 (1977): 868–871.
4. A. A. Maciejewski and C. A. Klein, "Obstacle Avoidance for Kinematically Redundant Manipulators," *International Journal of Robotics Research* 4, no. 3 (1985): 109–117.
5. T. Yoshikawa, "Analysis and Control of Robot Manipulators with Redundancy," in *Robotics Research: The First International Symposium*, ed. M. Brady and R. Paul (MIT Press, 1984), pp. 735–747.
6. O. Khatib and J. F. LeMaitre, "Dynamic Control of Manipulators Operating in a Complex Environment," in *Proceedings of the Third International CISM-IFTOMM Symposium (1978)*, pp. 267–282.
7. C. A. Klein and C. H. Huang, "Review of Pseudoinverse Control for Use with Kinematically Redundant Manipulators," *IEEE Transactions on Systems, Man, and Cybernetics* 13, no. 3 (1983): 245–250.
8. Y. Nakamura and H. Hanafusa, "Inverse Kinematic Solutions with Singularity Robustness for Robot Manipulator Control," *ASME Journal of Dynamic Systems, Measurement, and Control* 108 (1986): 163–171.